

SOUTH EASTERN UNIVERSITY OF SRI LANKA
SECOND EXAMINATION IN APPLIED SCIENCES - 2016 / 2017

SEMESTER II, JANUARY - 2019

MTM 22031 ELEMENTARY DIFFERENTIAL EQUATIONS

Time Allowed: One hour

Answer both questions. All questions carry equal marks.

1. (a) Solve the linear differential equation

$$\frac{dy}{dx} + \frac{y}{x(x+1)} = (x+1)e^{-x^2}.$$

- (b) Show that the differential equation

$$\frac{y}{x^2} + 1 + \frac{1}{x} \cdot \frac{dy}{dx} = 0$$

is not exact.

Verify that x^2 is its integrating factor of the equation and hence solve it.

- (c) Solve the differential equation

$$\frac{dy}{dx} = \frac{2x + 3y - 4}{4x + y - 3}$$

using suitable substitutions.

2. (a) A body at an unknown temperature is placed in a room temperature which is held at constant temperature 30°F. After 10 minutes the temperature of the body is 0°F and after 20 minutes the temperature of the body is 15°F. Temperature $T(t)$ of the body at time t is governed by the Newton's law of cooling:

$$\frac{dT}{dt} = k(T_{env} - T),$$

where T_{env} is the room temperature and k is a constant. Find the unknown initial temperature of the body.

- (b) Consider the initial value problem

$$\frac{dy}{dx} = xy, \quad y(0) = 1.$$

- (i) Construct the first three Picard's iterations.
(ii) Show that the sequence of iterations converges to the exact solution $y(x) = e^{\frac{x^2}{2}}$.

END OF THE PAPER