SOUTH EASTERN UNIVERSITY OF SRI LANKA

MTS 00033 MULTIVARIATE CALCULUS

ASSIGNMENT 2

Partial Differentiation

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1. Let $f(x,y) = 2x^2 - xy + 2y^2$. Using the definition of partial derivatives, evaluate
   $(a)\ \dfrac{\partial f}{\partial x}(1,2)$ and $(b)\ \dfrac{\partial f}{\partial y}(1,2)$.

2. Let $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2}; & if\ \ (x,y) \neq (0,0) \\ 0; & if\ \ (x,y) = (0,0) \end{cases}$.

   Compute $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$.

   Is f continuous at $(0,0)$.

3. Let $f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2}; & if\ \ (x,y) \neq (0,0) \\ 0; & if\ \ (x,y) = (0,0) \end{cases}$.

   Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

4. Find $\frac{\partial f}{\partial x}(0,1)$ and $\frac{\partial f}{\partial y}(0,1)$ for

$$f(x,y) = \sin x + y^2 \cos x + y^4 \tan^{-1}\big(x(y^2-1)\big) + \ln\big(2e^{\sin x} - 1\big) \sec xy \tan(y-1).$$

5. Find the directional derivatives of each of the following functions at the given points:
   i. $f(x,y,z) = x^2 z + y^3 z^2 - xyz$ in the direction of $-\underline{i} + 3\underline{k}$.
   ii. $f(x,y,z) = \sin yz + \ln x^2$ at $(1,1,\pi)$ in the direction of$(\, i + j - k\,)$.

6. Let $f(x,y) = x^3 - 3xy + 4y^2$. Find directional derivatives of f at the point $(1,1)$ in the direction of $\underline{u}$, where $\underline{u}$ makes a constant angle $\frac{\pi}{3}$ with the positive X axis.

7. If $f(x,y) = x^3 - 3xy + 4y^2$, find directional derivatives of f at the point $(1,1,1)$ in the direction of $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$.

8. Find the directional derivative for the function $f(x,y,z) = xz^2 - 3xy + 2xyz + 5y - 17$ from the point $(2,-6,3)$ towards the origin.

9. Let $f(x,y,z) = xz + e^{y-x^2}$.

   (a) Find the directional derivative of the function f at the point $(0,0,1)$ in the direction of $\big(0,1,\sqrt{3}\big)$.

   (b) Find the unit vector pointing in the direction along which $f(x,y,z)$ increases most rapidly at the point $(0,0,1)$.

10. Suppose that the pressure P over a surface is given by $P(x, y, z) = 5x^2 - 3xy + xyz$.
    i. Find the rate of pressure changes at $P(3,4,5)$
    ii. In which direction does the pressure increases fastest at P.
    iii. What is the maximum rate of change of the pressure at P.

11. Find the tangent plane to $f(x, y) = x^3y - 3xy^2$ at $(2, 1)$.

12. Find (x_0, y_0) so that the plane tangent to the surface $z = f(x, y) = x^2 + 3xy - y^2$ at $(x_0, y_0, f(x_0, y_0))$ is parallel to the plane $16x - 2y - 2z = 23$.

13. Find an equation of the tangent plane at $(1, 3)$ to the graph of $f\,(x, y) = xy^2 - xy + 3x^3y$.

14. Find an equation of the tangent plane at $(0, 3, -1)$ to the surface $f\,(x, y, z) \equiv ze^x + e^{z+1} + xy + y = 3$.

15. Let $f(x, y, z) = \sin(3x + yz)$. Show that $f_{xxyy} = f_{xyyx}$.

16. Show that $u(x, t) = \sin(x + \sin t)$ is a solution to the partial differential equation $u_t u_{xx} = u_x u_{tx}$.

17. Consider the function $f(x, y) = \begin{cases} \frac{x^2y^2}{x^2+y^2}; & \text{if } (x, y) \neq (0,0) \\ 0; & \text{if } x = 0 = y \end{cases}$. Show that $f_{xy}(0,0) = f_{yx}(0,0)$ however, f_{xy} is not continuous at $(0,0)$.