Chapter 01

Modeling Methodology

1.1 Introduction

In industry and commerce the availability of fast and powerful computers has made it possible to 'mathematize' and 'computerize' a range of problems and activities previously unsolvable owning to their complexity. Therefore the application of mathematics and statistics have therefore increased over the last 25 years. These opportunities can only be met if there are enough qualified professional available with the right qualities to contribute.

Professional modelers have to deal with variety of real problems, and their main task is to translate each problem into a mathematical form. The power industry provides many examples of how mathematical modelling is used. Problems of flow of water, electricity, gas and oil, and the necessity to match provision of these with varying demand, clearly lend themselves to mathematical treatment.

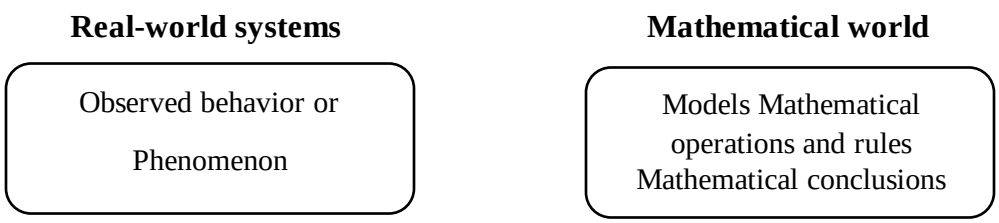
Mathematical modelling plays an important part are planning, security, and communication. To understand our world, we often try to describe a particular phenomenon (spring mass system ; Interest money in bank ; Lanchester combat models) mathematically by means of a functions an equation or a set of inequalities. Such an idealization of a real-world scenario is called a mathematical model.

In modelling our world we are interested in predicting the value of the variable in the future. A good mathematical model can help us to understand the behavior better and plan to the needs for the future.

Every branch of knowledge has two aspects; one is theoretical and other is experimental and observational.

1.2 Definition (Mathematical Modelling)

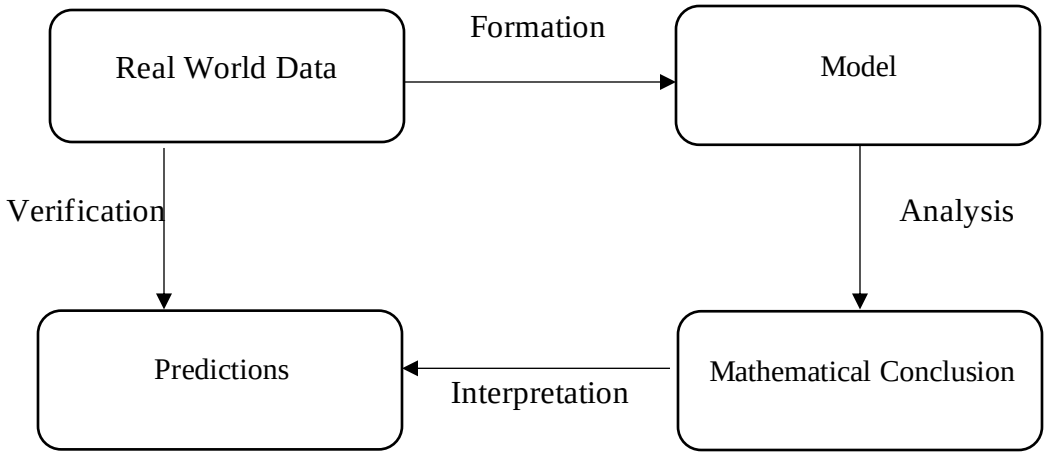
The process by which we use mathematical expressions to describe a real quantitative situation is called modeling. Modeling consists of writing in mathematical terms what is first expressed in words, using variables where necessary. The preceding example illustrates modeling: even though we do not know the salary of the individual, we were nonetheless able to obtain an expression that adequately represents the taxes paid by him.



1.3 Creating a mathematical model

- We are given a word problem
- Determine what question we are to answer
- Assign variables to quantities in the problem so that you can answer the question using these variables
- Derive mathematical equations containing these variables
- Use these equations to find the values of these variables
- State the answer to the problem

1.4 Process of mathematical modeling



Example No: 01

A square bottom box without cover is made from a material that costs $ 0.75 per square meter for the sides and $ 0.95 per square meter for the bottom. Express the total cost of the material required to construct the box in function of its width and height.

Example No: 02

The three phases of a project must be undertaken sequentially, which means that one phase cannot begin before the previous phase is finished. We know that the cost of each of the phases breaks down into a fixed cost, independent of its duration, and a variable cost, which depends on the duration. The following table summarizes the situation:

PHASE	**1**	**2**	**3**
FIXED COST	318 000 $	212 000 $	220 000 $
VARIABLE COST	15 000 $ / day	14 000 $ / day	16 000 $ / day

The designer of the project must propose a price for the project. He would like to set a price that ensures a profit margin of at least 10%. Express the total cost of the project and the price the designer should propose in function to the duration of each phase.

Example No: 03

A farmer is looking to divide to plant different cultures. Traditionally, corn fields returned $ 3.50 $ per square meter. Oat fields returned $ 2.75 per square meter. Orchards produced revenues of $ 4.50 per square meter. The farmer has a land of 1 million square meters. In order to feed his farm animals, the cultivator must dedicate a minimum of 300 000 square meters to the culture of corn and oats (together). However, since corn is more susceptible to long periods of drought, he does not want this culture to occupy more than 200 000 square meters. Lastly, he would like to allot the same amount of space to oats and orchards.

Which expression correctly represents the revenues of the farmer? Model all constraints that the farmer must respect.

Example No: 04

The effectiveness of car wind screen single bent wiper, here the investigator requires as expression for the area of the window wiped by the following blade.

Description	Symbol	Unit
Breath of the rear window	b	cm
Height of the rear window	h	cm
Length of the blade	r	cm
Length of arm of the wiper	L	cm
Angle of rotation of arm	α	
Angle between bent blade and wiper arm	β	
Distance of wiper mounting form lower edge of window	d	cm
Lateral distance of mount form center of window	x	cm
Area of coverage of glass wiped	A	cm

The following assumptions will be introduced for this investigation;

I. The wiper mount will be taken at the base of the window; hence referring to the variable list, d=0.
II. The shape of the window will be taken as rectangular
III. The wiper arm executes symmetric sweeps of $\alpha/2$ degrees either side of the vertical.

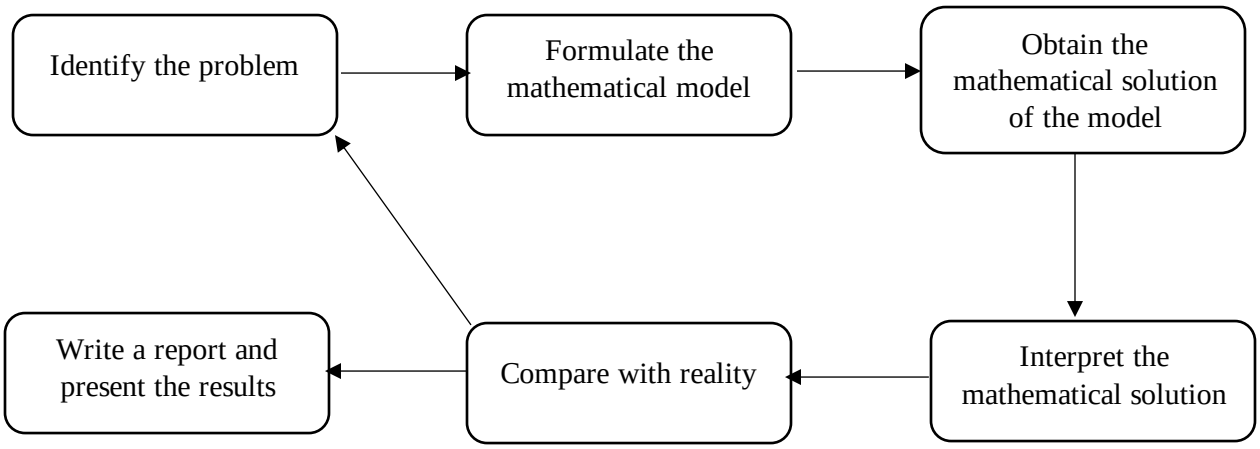
Example No: 05

How many cars can pass through a set of traffic lights when they turn green for a period of 15 seconds?

The following assumptions will be introduced for this investigation:

(a) The junction is not blocked any way.
(b) All cars intend to pass directly across the junction.
(c) All vehicles are cars of the same size, 5m in length and initially at rest.
(d) There is a 2m gap between each stationary car.

1.5 Modelling Methodology



Identify the real problem:

What do we want to Know? What is the purpose and objective? How will the outcome be judged? What are the sources of facts and data, and are they reliable? Is there one particular unique answer to be found? Classify the problem: is it essentially deterministic, or stochastic? Do we need to use simulation?

Formulate a mathematical model:

Look first for the simplest model. Draw diagrams where appropriate. Identify and list the relevant factors. Collect data and examine them for information explaining the behaviour of the of the variables. Collect more data if necessary. Denote each variable by an appropriate symbol and assign units. State any assumptions that you decide to make. Draw up relations and equations connecting the problem variables, using your mathematical skills, e.g. Proportionality, linear and non-linear relations, empirical relations, input-output principles, Newton's laws of

motion difference and differential equations, matrices, probability, statistical distributions, etc. (See chapter 5 for some help with this stage.)

Obtain the mathematical solution of the model

Employ algebraic or numerical methods, as well as calculus and graphs. If appropriate, write computer applications or use a ready-made package.

Interpret the mathematical solution

Examine the results obtained from the mathematics. Have the values of the variables got the correct sign and size? Do they increase or decrease when they should? Should a certain graph be linear? Consider large and small values of the variables to check for sensible behavior. Have you got the 'best' solution that you expected or should some initial conditions ne changed?

Compare with reality:

Can your results be tested against real data? Do your mathematical solutions make sense? Do your predictions agree with the real data? Evaluate your model. Has it fulfilled its purpose? Can the model be significantly improved by greater mathematical sophistication? Do the interim results suggest that more accuracy is needed by rerunning with an improved model? If yes then go to initial step; otherwise write the report.

Write a report:

Who is the report for and what do the readers want to know? How much details is required in the report? How can we construct the report so that the important features are clear and the results that we want to be read stand out?

Example No: 06

It is about rain; you have to walk a short distance of about 1 km between home and college as there some hurry, you do not bother to take rain court or umbrella but decide to change it suppose that it now starts to rain heavily and you do not turn back how much wet will you get?