Chapter 03

Solving Mathematical Model Using Difference Equation

3.1: Difference Equation

A difference equation is a relation between a finite member of a sequence. The members of the sequence $\{x_n\}_{n=1}^{\infty}$ are x_n,x_{n-1},and x_{n-2}.

If $\{x_n\}_{n=1}^{\infty}$ is a sequence relation $f(x_1,x_2,\dots,x_n) = 0$ is a difference equation, where f is any function.

Ex: $x_n = x_{n-1} + x_{n-2}$

3.2: Order of the Difference Equation

Then the order of the difference equation is the difference between the highest and lowest index of the equation.

Ex: $x_n = x_{n-1} + x_{n-2}$

Order = $n - (n-2) = 2$.

3.3: Financial Models

Example No: 01

The production of an industry is represented by P_n. Find the model for the following cases and solve using initial values as P_0.

a. The production double every year.

b. The production growth rate is increased by 25%.

Example No: 02

Assume that your grandparents have an annuity, the value of the annuity increases each month by an automatic deposit of 1% of interest on the previous month's balance. Your grandparents withdraw \$ 1000 at the beginning of each month. Currently they have \$ 50000 for living expenses in the annuity. Model the annuity with a dynamic system. When will the annuity become out of money?

3.3.1 Simple Interest

When only the principle earns interest for the entire period of the transection the interest due at the end of the time period is called "Simple Interest".

$p -$ principle value

$t -$ time period in years

$r -$ interest rate

$I -$ interest earned in a unit of time for one unit of principle.

Then total amount $S = p + I$, where $I = prt$, Then total amount after t periods is

$$S = p(1 + rt).$$

Example No: 03

Mr. Rohan invested Rs. 12800 in two different investment plans, A and B, at respective simple interest rates of 11% and 14%. What was the value of plan B if the interest earned over the course of two years was Rs. 3508?

Example No: 04

A lender claims to be lending at simple interest, but he adds the interest every 6 months in the calculation of the principal. The rate of interest charged by him is 8%. What will be the effective rate of interest.

3.3.1 Compound Interest Method (CIM)

At the stated intervals the interest is added to the principal under CIM. In this situation we say that the interest is compounded into the principal and therefore we have

I. Principal p_t increase periodically
II. The interest compounded into the principle increases periodically,
III. Compound amount = The sum due at the end of the transection
IV. Compound Interest = (Compounded amount) – (Original principal)

Example No: 05

1000 dollars in initial principal for a three years period at 5% interest. Calculate the compound interest.

3.4 Matrix models

Example No: 06

Let $A = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. find the matrix model A^n , where $n \in z^+$.

3.4 Population Models

Example No: 07

A population of herbivorous living in a certain area is declining by 5% every year. As their number is declining, they leave more vegetation. So, more animals move into that area. Assume that 300 animals move into the area per year. Derive and solve the model.

Example No: 08

Suppose a certain population of owls is growing at the rate of 2% per year. Let X_0 represent the size of the initial population of owls and X_n the number of owls n years later. Find X_4 in relation to X_0. Create a discrete model to depict owl population increase.Assume there were initially 100 owls. Calculate the owl population after (i) 20 years and (ii) 150 years.

Example No 09

At the moment, a lake includes 10,000 fish. Without fishing, the population of fish would expand by 15% per year. It is proposed that fishing be permitted at a pace of 2000 fish each year. Model the lake's fish population and evaluate your results,

What happens to the fish population in the end?

3.6 Mixing Problem Models

Example No: 05

Suppose a lake contains 100 000 m^3 of water with 5% pollution by volume. Every day 1000 clean water flows into the lake and 1000 m^3 of polluted lake water flows out. How long will it take for the pollution in the lake to drop to a safe level of 1% ?

Example No: 06

Assume that the pollution continues to be added to this lake while 995 $m^3$of clean water flow in every day and the outflow from the lake is 1000 m^3 per day. Find the model for p_n and solve the model?

Example No: 07

Suppose water from the first lake flows into a second lake, and the daily out flow is 1200 m^3while 200 $m^3$of clean water also flows in every day from stream. Write the model for pollution level?