Chapter 04

Solving Mathematical Model Using Differential Equation

4.1 Unlimited population growth

Let's assume that we are aware of the population size P_o at $t_o = 0$. We want to be able to anticipate the population P at some point in the future, t_1. When the population size is big and we take into account the change in the population, P, over a brief period of time, t, this will not be a severe error.

The purpose of our model will be to predict the future population, i.e to give $P(t)$ as a function of $t.$ So simple model for the change in $P(t)$ during a short period of time Δt is

{Increase in population during time Δt} = {births during time Δt}-{deaths during time Δt}+ {immigration during time Δt}- {emigration during time Δt}

Example No: 01

Suppose that b > 0 is the per capita average birth rate, and m > 0 the per capita average mortality rate. Find the population after t years assuming initial population as y_0.

Example No: 02

The population of a town grows at a rate proportional to the population present at time t. The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?

Example : 03

A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors, the population will triple in two weeks of time. On any given day, there is a net migration of 15 insects into the area, 16 are eaten by the local bird population and 7 die due to natural causes. If there are initially 100 insects in the area, will the population survive? If not, when do they die out?

4.2 Growth and Decay

The initial – value problem

$$\frac{dx}{dt} = kx, \;\; x(t_0) = x_0$$

Where k is a constant of proportionality, serves as model for diverse phenomena involving either growth or decay. We saw in Section 1.3 that in biological applications the rate of growth of certain populations (bacteria) over short period of time is proportional to the population present at time t.

Example No: 04

A culture initially has p_0 number of bacteria. At $t = 1h$ the number of bacteria is measured to be $\frac{3}{2}\, p_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t, determine the time necessary for the number of bacteria to triple?

Example No: 05

A breeder reactor converts relatively stable uranium-238 into the isotope plutonium 239. After 15 years it is determined that .043% of the initial amount A_0 of plutonium has disintegrated. Find the half-life of this Isotope if the rate of disintegration is proportional to the amount of remaining?

4.3 Mixing problem

Example No: 06

A large vat holds 100 gallons of water that is to be mixed with sugar and then used to make soft drinks. Sugar-water containing 5 tablespoons of sugar per gallon enters the vat through a pipe at a rate of 2 gallons per minute. Another pipe pumps sugar-water with 10 tablespoons of sugar per gallon into the vat at a rate of 1 gallon per minute. The vat is kept well mixed, so that the concentration of sugar in the vat is essentially uniform. Sugar-water is drained out of the vat at a rate of 3 gallons per minute. Notice that the amount flowing into the vat is the same as the amount flowing out of the vat, so there are always 100 gallons of liquid in the vat. Find the amount of sugar in the vat at time t if the vat initially has 900 tablespoons in it.

Example No: 07

A tank has pure water flowing into it at 10 l/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 l/min. Initially, the tank contains 10 kg of salt in 100 l of water. How much salt will there be in the tank after 30 minutes?

4.4 Motion Under gravity

Example No: 08

Set up the model that give the velocity of a 60kg sky driver that jumps out of a plane with no initial velocity and air resistance of 0.8V

Example No: 09

A particle falls under gravity in a medium in which resistance is not negligible. Find the distance travelled by a particle at a time before it hits the ground and the velocity at that point.

4.4 Newton's Cooling Law

Let's suppose we put an object in a medium whose temperature is different from the object's temperature. The rate at which an object or entity changes its temperature in anticipation of radiation is explained by Newton's law of cooling. This change is almost proportional to the difference between the object's temperature and its surroundings' temperature, given that this difference is quite small. The model can be described as

$$\frac{dT}{ds} \propto (T - T_{env}),$$

Where T-Object Temperature

T_{env}-Surrounding Temperature

Example No: 10

At 12:30 a.m., police showed up at the scene of a murder. They quickly take the body's temperature, which is 90°F, and record it. They also carefully inspect the area. It is two in the morning when they are done with the inspection. They take the body temperature again, which has dropped to 85°F. The temperature at the crime scene has been consistent at 80°F. Determine the time of the murder.