SOUTH EASTERN UNIVERSITY OF SRI LANKA
Faculty of Applied Sciences
Department of Mathematical Sciences

MTC 12011 MATHEMATICS FOR BIOLOGY II

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1. INTRODUCTION TO MATRICES

1.1 Basic Definitions

Definition 1.1 A matrix (for our purpose) of size $m \times n$ is an array of numbers of the form

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m\times n}.$$

i^{th} row of the matrix is $(a_{i1} \quad a_{i2} \quad \cdots \quad a_{in})$ and j^{th} column of it is $\begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{pmatrix}$.

The value $a_{ij};\ 1 \le i \le m,\ 1 \le j \le n$ is called the entry (element) in the i^{th} row and j^{th} column.

We denote a matrix by a capita letter and its elements by the corresponding lower case letter. That is, $A = (a_{ij}),\ B = (b_{ij})$, etc.

Example 1.1 Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 11 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 6 \\ 5 & 1 \\ -1 & 2 \end{pmatrix}$.

Note that A and B are not same in size. A is a 2×3 matrix; B is a 3×2 matrix.

$a_{21} = 0, \qquad b_{21} = 5.$

Definition 1.2 Two matrices $A = (a_{ij})$ and $B = (b_{ij})$ are equal if and only if they are same size and $a_{ij} = b_{ij}$ for all i, j.

1.2 Types of Matrices

Definitions 1.3

1. If $m = 1$, then the matrix is called a row matrix (or row vector).

   Examples: $(2, 3)$, $(3, 4, 5, 7)$

2. If $n = 1$, then the matrix is called a column matrix (or a column vector).

   Examples: $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ -1 \\ 5 \end{pmatrix}$.

3. If $m = n$, the matrix is called a <u>square matrix</u>. Let $A_{n\times n} = (a_{ij})$ be a square matrix of size $n \times n$. We simply write this as A_n.

Examples: $A = \begin{pmatrix} 1 & 0 \\ 3 & 9 \end{pmatrix}$ is a square matrix of order 2.

$B = \begin{pmatrix} 1 & 0 & 7 \\ -1 & 3 & -2 \\ 4 & 5 & 1 \end{pmatrix}$ is a square matrix of order 3.

4. The $n -$ tuple $(a_{11}, a_{22}, \cdots, a_{nn})$ is called the (main) <u>diagonal</u> of a square matrix A.

In the above examples, diagonal of A is $(1, 9)$ and that of B is $(1, 3\ 1)$.

5. The trace of a square matrix is defined as the sum of the main diagonal elements. That is, $\text{tr}(A) = \sum_{i=1}^{n} a_{ii}$.

In the above examples, trace of A is $1 + 9 = 10$ and that of B is $1 + 3 + 1 = 4$.

6. A square matrix A is said to be a <u>diagonal matrix</u> if $a_{ij} = 0$ for all $i \neq j$ and may be denoted by $A = \text{diag}(a_{11}, a_{22}, \cdots, a_{nn})$.

Examples: $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$.

7. A diagonal matrix with $a_{ii} = 1$ for all $i = 1, 2, \cdots, n$ is called an <u>identity matrix</u> and is denoted by I_n, simply I. i.e. $I = \text{diag}(1,1, \cdots ,1)$

Examples: $I_2 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ identity matrix of order 2,

$I_3 = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ identity matrix of order 3.

8. If entries above the main diagonal are only zeros, then the matrix is lower triangular and if the entries below the main diagonal are only zeros, then it is upper triangular. That is, a square matrix is said to be <u>lower triangular</u> if $a_{ij} = 0$ for $i < j$ while it is said to be <u>upper triangular</u> if $a_{ij} = 0$ for $i > j$.

Examples: $\begin{pmatrix} 3 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & 2 & 0 \end{pmatrix}$ is a lower triangular matrix.

$\begin{pmatrix} 2 & 0 & -7 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & -5 \end{pmatrix}$ is an upper triangular matrix.

9. A matrix is said to be <u>zero matrix</u> (or null matrix) if $a_{ij} = 0$ for all i, j. The zero matrix is denoted by $\mathbf{0}$.

Examples: $\mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $\mathbf{0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\mathbf{0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ are null matrices.

EXERCISES

1. Let $A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 4 & 1 \end{pmatrix}$.

   What is the size of the matrix A?

   Compute $a_{23} \times a_{12} + a_{22}/a_{21}$.

2. Let's say you're in avid reader and you read fiction, non-fiction books, and magazines, both in paper copies and online. You want to keep track of how many different types of books and magazines you read in a month, and store that information in matrices. The rows represent different types of books: friction, non-friction and magazines. The columns represent the source type: Paper and online. Your matrix is as follows:

$$B = \begin{pmatrix} 2 & 5 \\ 4 & 3 \\ 1 & 4 \end{pmatrix}.$$

   What is the size of B?

   How many online magazines you read in that particular month?

   How many total paper books you read?

3. Let $P = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & 3 \\ 3 & 4 \\ 4 & 1 \end{pmatrix}$, $R = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 5 & 4 \\ 1 & -4 & 3 \end{pmatrix}$, $S = \begin{pmatrix} 0 & 4 & 3 \\ -4 & 0 & 5 \\ -3 & -5 & 0 \end{pmatrix}$ and,

   $T = \begin{pmatrix} 5 & 4 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 7 \end{pmatrix}$.

   i. Classify them in to square matrices, non-negative matrices, positive matrices, lower triangular matrices and upper triangular matrices.

   ii. List out the matrices of same size.

   iii. Find the trace of the matrix R.

   iv. If $Q = (q_{ij})$, write the matrix $U = (q_{ji})$. What is the size of U?

   v. For a matrix given above, $a_{ij} = -a_{ji}$. What is that matrix?