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MTC 12011 MATHEMATICS FOR BIOLOGY II

2019 / 2020

2 MATRIX OPERATIONS

Definition 2.1 (Matrix Addition) If $A = (a_{ij})$ and $B = (b_{ij})$ are matrices of the same size $m \times n$, then the sum of A and B is the matrix of the size $m \times n$ defined by $C = A + B$, where

$$c_{ij} = a_{ij} + b_{ij} \quad \text{for all } i, j.$$

Definition 2.2 (Scalar Multiplication) Let $A = (a_{ij})$ be any matrix and α be any real number (scalar). Then, the scalar multiplication of A is defined by $B = \alpha A$, where

$$b_{ij} = \alpha\, a_{ij} \quad \text{for all } i, j.$$

Remark2.1: The size of αA is as same as size of A.

Remark2.2: We define the difference $D = A - B$ by $d_{ij} = a_{ij} - b_{ij}$ for all i, j.

Example 2.1 Let $A = \begin{pmatrix} 5 & 3 & 1 \\ 0 & 1 & 4 \\ -2 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 7 \\ 0 & 2 & -1 \\ -2 & 5 & 0 \end{pmatrix}$. Compute $\frac{1}{2}(2A - 3B)$.

Theorem 2.1 Let A, B, C be any three matrices of the same size and α, β be any two real numbers. Then,

(a) Closure property: $A + B$ is also a matrix of the same size and is unique.

(b) Associativity: $(A + B) + C = A + (B + C)$.

(c) Commutativity: $A + B = B + A$.

(d) Distributive laws: $(\alpha + \beta)A = \alpha A + \beta A$,

$$\alpha(A + B) = \alpha A + \alpha B,$$

$$\alpha(\beta A) = \alpha\beta A.$$

(e) $0\, A = \mathbf{0}$.

(f) $\alpha\, \mathbf{0} = \mathbf{0}$.

Definition 2.3 (matrix Multiplication) Let $A = (a_{ij})$ be $m \times p$ matrix and $B = (b_{ij})$ be an $p \times n$ matix. Then the product $C = AB$ is an $m \times n$ matrix defined by

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}\,, \qquad 1 \le i \le m, \;\; i \le j \le n.$$

Remark2.3: Note that number of columns in A is equal to the number of rows in B. In this case, we say that A and B are conformable for the product AB.

Example 2.2 Let $A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 0 & 1 \\ -2 & 1 & 1 \end{pmatrix}$. Compute AB or BA which is conformable for the matrix multiplication.

Example 2.3 Let $A = (2 \quad -1)$ and $B = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. Compute AB and BA.

Example 2.4 Show that $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix} = \mathbf{0}$.

Remark2.4: Note that if the product of two matrices is zero, then one of them need not to be zero matrix.

Example 2.5 Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$. Find A^2 and A^3.

Theorem 2.2 Let A, B, C be matrices for which all oprations below make sense. Then

(a) Associativity: $(AB)C = A(BC)$.

(b) Distributive laws: $A(B + C) = AB + AC,\ A(B - C) = AB - AC,$

$(\alpha A)B = A(\alpha B) = \alpha(AB),$

$(\alpha A)(\beta B) = \alpha\beta(AB)$

(c) $\mathbf{0}\,A = \mathbf{0}A = \mathbf{0}.$

Remark2.5: Note that matrix multiplication is not commutative. That is, $AB \neq BA$ in general.

Definition 2.4 We define the transpose of a matrix A of size $m \times n$, and denoted by A^T, to be the $n \times m$ matrix with entries $(A^T)_{ij} = a_{ji}$.

Remark2.6: In other words, the transpose of a matrix is obtained by interchanging the rows and columns of the given matrix.

Theorem 2.3 Let A, B be two matrices. Then,

(a) $(A^T)^T = A$.

(b) $(A \pm B)^T = A^T \pm B^T$.

(c) $(AB)^T = B^T A^T$.

(d) $(cA)^T = c\,A^T$.

Example 2.6 Verify the theorem 2.3 for the matrices $A = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 3 \\ 3 & 1 \end{pmatrix}$ and $c = 5$.

Definition 2.5 A matrix X is called symmetric if $X^T = X$ and skew symmetric if $X^T = -X$.

Example 2.7 Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$. Show that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric.