SOUTH EASTERN UNIVERSITY OF SRI LANKA
Faculty of Applied Sciences
Department of Mathematical Sciences

MTC 12011 MATHEMATICS FOR BIOLOGY II

2019 / 2020

---

4 Invertible Matrices

Definition 4.1 The matrix A is said to be <u>invertible</u> (non-singular) if there is a matrix Bsuch that

$$AB = BA = I.$$

In this case B is called the inverse of A and is denoted by A^{-1}.

Example 4.1 Let $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 3 & -1 \\ -2 & 3 & -1 \end{pmatrix}$. Verify that $A^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 3 & -2 \\ -3 & 7 & -5 \end{pmatrix}$.

Theorem 4.1 Let A, B be two invertible matrices and α is a scalar. Then,

(a) αA is invertible and $(\alpha A)^{-1} = \frac{1}{\alpha} A^{-1}$.

(b) $(AB)^{-1} = B^{-1}A^{-1}$, provided AB, BA are defined.

(c) $(A^{-1})^{-1} = A$.

(d) $(A^{T})^{-1} = (A^{-1})^{T}$.

Theorem 4.2 A matrix is invertible (nonsingular) if and only if $\det(A) \neq 0$.

Definition 4.2 Adjoint

The adjoint of a square matrix A is the transpose of its cofactor matrix and is denoted by $\operatorname{adj}(A)$.

Theorem 4.3 If A is non-singular, then

$$A^{-1} = \frac{1}{\det A} \operatorname{adj}(A).$$

In particular, inverse of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

provided $ad - bc \neq 0$.

Example 4.2 Find the inverse of the matrices

$(\boldsymbol{i})$ $A = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$, (ii) $B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 2 & 1 \end{pmatrix}$.