SOUTH EASTERN UNIVERSITY OF SRI LANKA
Faculty of Applied Sciences
Department of Mathematical Sciences

MTC 12021 MATHEMATICS FOR BIOLOGY II

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5. System of Linear Equations

Consider a system of such n linear equations in n variables

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_{n.}$$

This can be written in matrix form as

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

By denoting the coefficient matrix $A = (a_{ij})$, $X = (x_1, x_2, \cdots, x_n)^T$ and $b = (b_1, b_2, \cdots, b_n)^T$, we can written the system as

$$AX = b.$$

Definition 5.1 By <u>solution</u> of the system, we mean a sequence of scalar values $x_1 = c_1$, $x_2 = c_2, \cdots$, $x_n = c_n$ satisfying all equations of the system.

If a solution does exist, the system is said to be <u>consistent</u>.

A system which has no solution is called <u>inconsistent</u>.

To solve a system of linear equation $AX = b$:

$$A^{-1}(AX) = A^{-1}b$$

$\Longrightarrow \quad X = A^{-1}b.$

Example 5.1 Solve the system of equations

$$x + 2y + 3z = 18,\ 2x + 3y + z = 13,\ 3x + 2y + z = 6.$$

Theorem 5.1 Cramer's Rule

Let $AX = b$ be a system of linear equations in n variables. suppose that the coefficient matrix A is invertible. Then, the solution of the system is given by

$$x_j = \frac{\det(A_j)}{\det(A)}, \qquad j = 1, 2, \cdots, n$$

where A_j is the matrix formed by replacing j^{th} column of A with b.

Example 5.2 Solve the following system equation by Cramer's rule:

$$2x + y + z = 6, \qquad 3x + 2y - 2z = -2, \qquad x + y + 2z = 4.$$

EXERCISES

1. Verify that if $A = \begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix}$, then $A^{-1} = \begin{pmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{pmatrix}$.

   Solve the system

$$7x + 2y + z = 8,$$
$$3y - z = -1,$$
$$-3x + 4y - 2z = -5.$$

2. Compute the inverse of the matrix $\begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix}$.

   Hence solve the system of linear equations

$$x + 4y + 3z = 12,$$
$$-x - 2y = -12,$$
$$2x + 2y + 3z = 8.$$

3. Compute the inverse of the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}$.

   Hence solve the system of linear equations

$$x + 3z = 1,$$
$$3x + 3y + 4z = 12,$$
$$2x + 2y + 3z = 1.$$

4. Use Cramer's Rule to solve

$$x + 2y + z = 5,$$
$$2x + 2y + 4z = 6,$$
$$x + 2y + 3z = 9.$$

5. Use Cramer's Rule to solve

$$2x + y - 3z = 0,$$
$$4x + 5y + z = 8,$$
$$-2x - y + 4z = 2.$$